

# **ROLE OF DISTRIBUTED INTER-BRISTLE FRICTION FORCE ON BRUSH SEAL HYSTERESIS**

Helen Zhao and Robert Stango  
Marquette University  
Milwaukee, Wisconsin



## **Role of Distributed Inter-bristle Friction Force On Brush Seal Hysteresis**

---

By

**Helen Zhao and Robert Stango**

Department of Mechanical and Industrial Engineering  
Marquette University, Milwaukee, WI  
Email: [haifang.zhao@marquette.edu](mailto:haifang.zhao@marquette.edu) ; [robert.stango@marquette.edu](mailto:robert.stango@marquette.edu)



# Introduction and Background

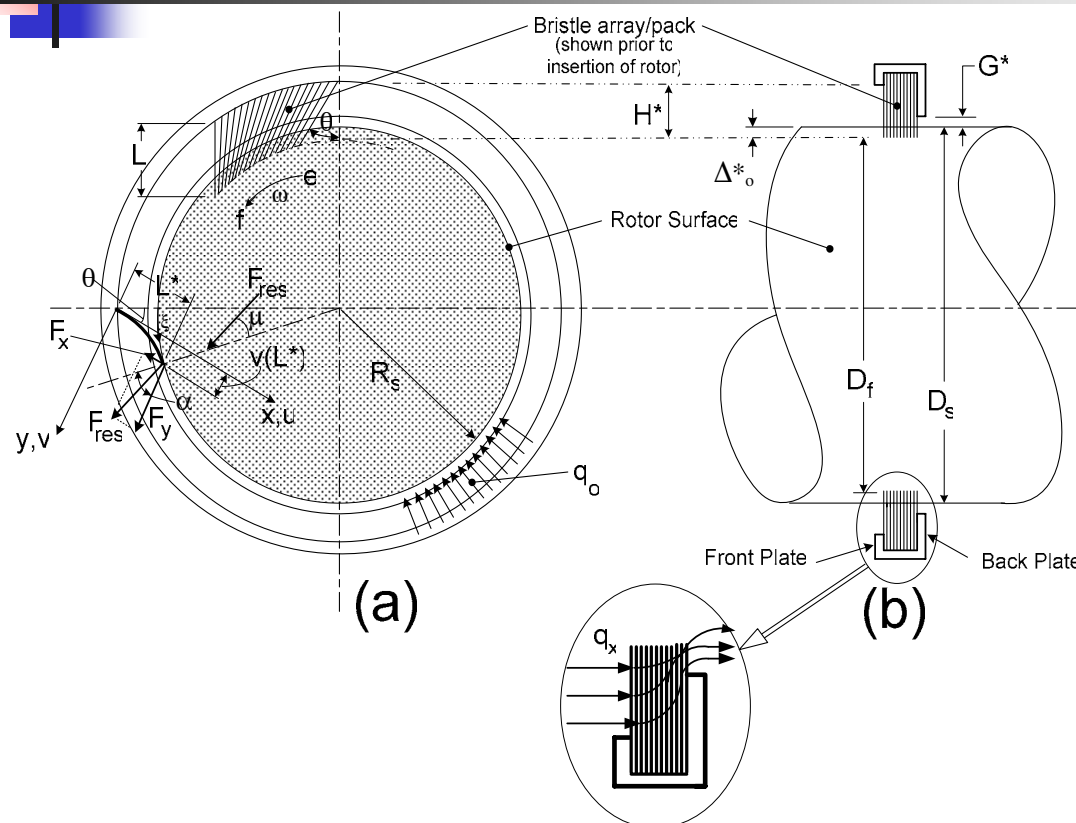


Figure 1 Brush seal with various working loads

Figure 1

- Interference parameter  $\Delta_o^*$
- Inward radial flow-induced load  $q_o$
- Contact force  $F_{res}$  generated at interface of fiber tip and rotor
- Local oncoming flow of gas toward bristle pack  $q_x$

# Inter-bristle friction force model

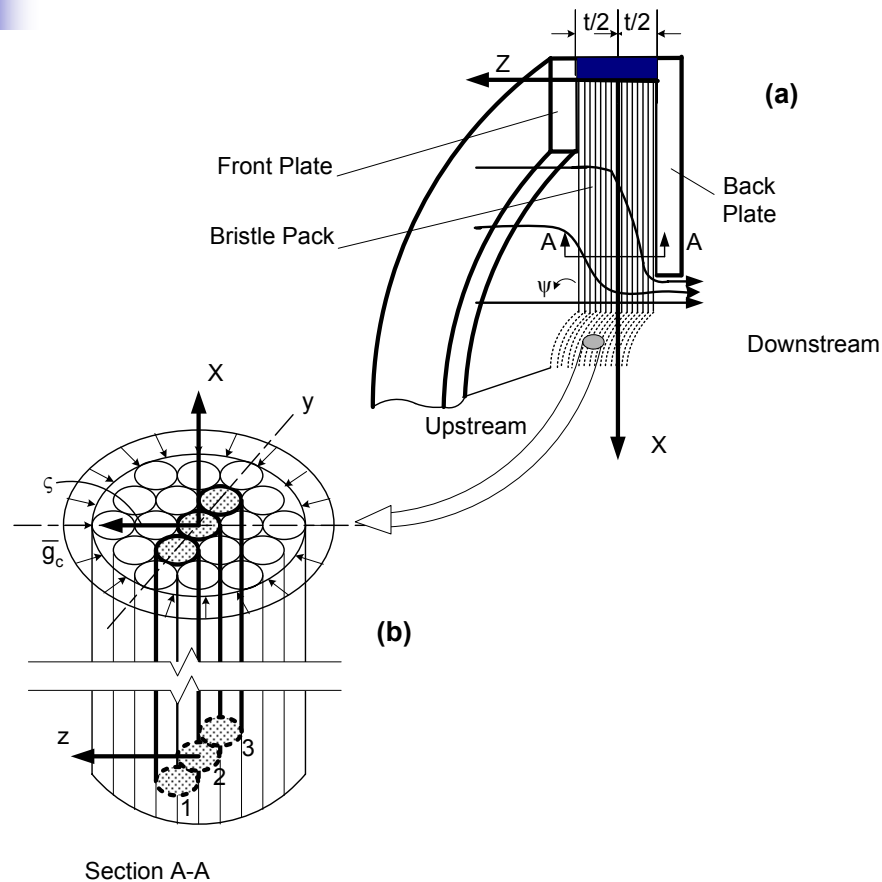


Figure 2

- (a) Depiction of partial brush seal with front and back plate that constrain bristle pack
- (b) Section A-A view, depicting the compactive load  $g_c$  around bristle pack. The interactive forces of three fibers (1, 2, 3) are studied for hysteresis phenomenon

# Inter-bristle Friction Model (cont'd)

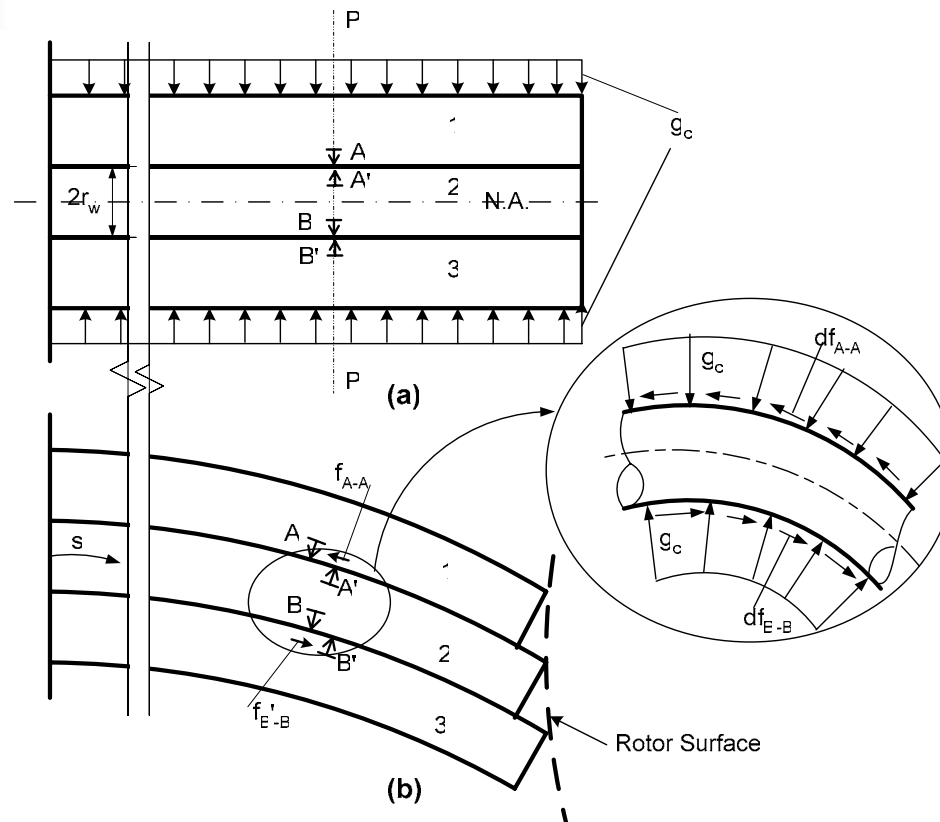
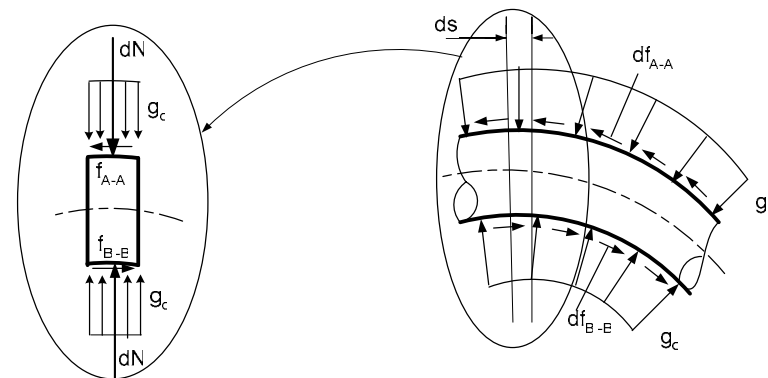


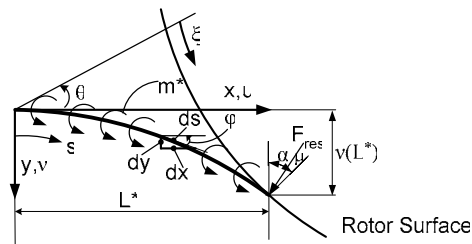
Figure 3

- Three un-deformed neighboring fibers subjected to the compactive load  $g_c$
- Deformation of fibers under compactive load  $g_c$ .

# Inter-bristle Friction Model (cont'd)



(a)



(b)

Figure 4

(a) Segment of the deformed fiber subjected to the uniform compressive load  $g_c$  and traction force  $f_{A-A}$  and  $f_{B-B}$

(b) simplified model depicting interaction between neighboring bristles as uniformly distributed moment  $m^*$  along deformed fiber

# Inter-bristle Friction Model---derivation of $m^*$

Refer to Fig.3 and Fig.4:

1. Differential frictional force  $df_{A-A'}$  and  $df_{B'-B}$

$$\left. \begin{aligned} df_{A-A'} &= \mu dN; df_{B'-B} = \mu dN \\ dN &= ds \cdot g_c \end{aligned} \right\} \Rightarrow \begin{aligned} df_{A-A'} &= \mu g_c ds \\ df_{B'-B} &= \mu g_c ds \end{aligned}$$

2. Differential moment  $dm$ :

$$dm = 2\mu g_c r_w ds$$

3. Resisting bending moment  $m$

$$m = 2\mu g_c r_w L$$

4. Distributed bending moment per unit length  $m$

$$m^* = 2\mu g_c r_w$$

If Hexagonal closed-pack, then  $m^* = 2\mu g_c r_w (1 + 2 \cos 60)$



# Governing Equation

According to Euler-Bernoulli Law:

$$EI\kappa = M_{m^*} + M_{F_{res}} \quad \text{with} \quad \kappa = \frac{d\phi}{ds}$$

Governing Equation:

$$EI \frac{d^2\phi}{ds^2} = m^* - F_{res} \cos(\alpha - \mu - \phi)$$

Non-dimensional form of governing equation:

$$\frac{d^2\phi}{ds^{*2}} = \frac{m^* H^{*2}}{EI} - \frac{F_{res} H^{*2}}{EI} \cos(\alpha - \mu - \phi)$$



# Boundary conditions and constraint conditions

## Boundary conditions:

1. slope constraint at the bristle origin, i.e.  
 $\Phi=0$  at  $s=0$

2. Free of moment at the bristle tip, i.e.  
 $d\Phi/ds=0$  at  $s=L$

## Constraint conditions:

$$\left| x_t - x_\xi \right| < \varepsilon; \left| y_t - y_\xi \right| < \varepsilon$$

Where,

$$x_t = \int_0^L \cos \phi ds; y_t = \int_0^L \sin \phi ds$$

$$x_\xi = (R_s + H^* - \Delta_o^*) \cos \theta - R_s \cos(\theta + \frac{\xi}{R_s})$$

$$y_\xi = R_s \sin(\theta + \frac{\xi}{R_s}) - (R_s + H^* - \Delta_o^*) \sin \theta$$



# Eccentricity of Shaft

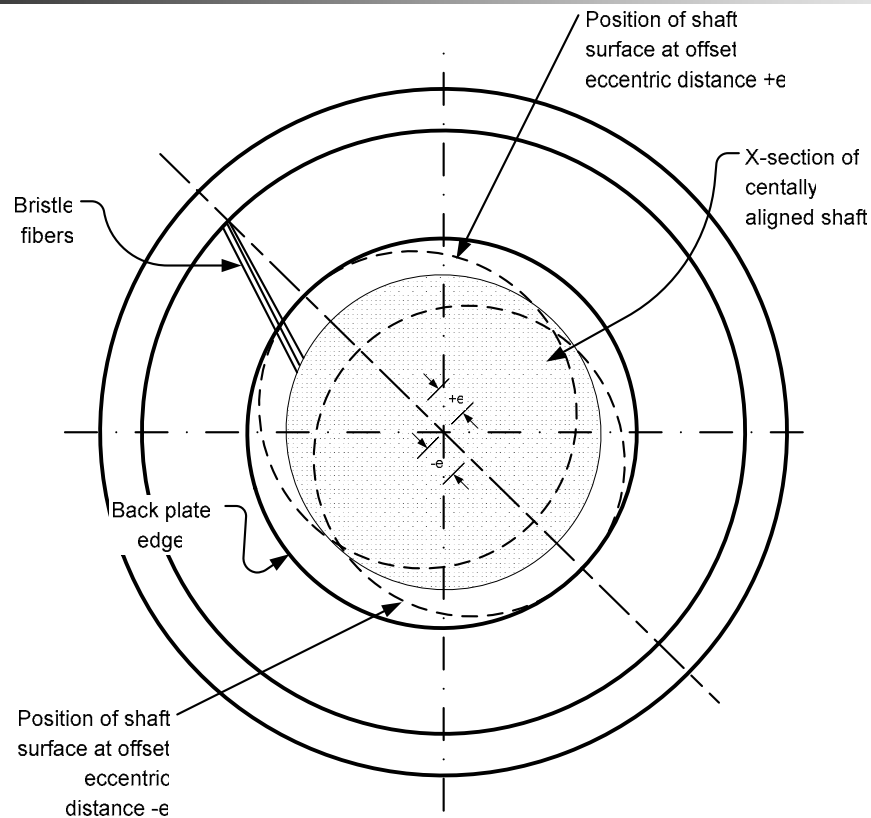


Figure 5

Eccentric movement of rotor and displacement of bristle pack

# Numerical Results and Discussion

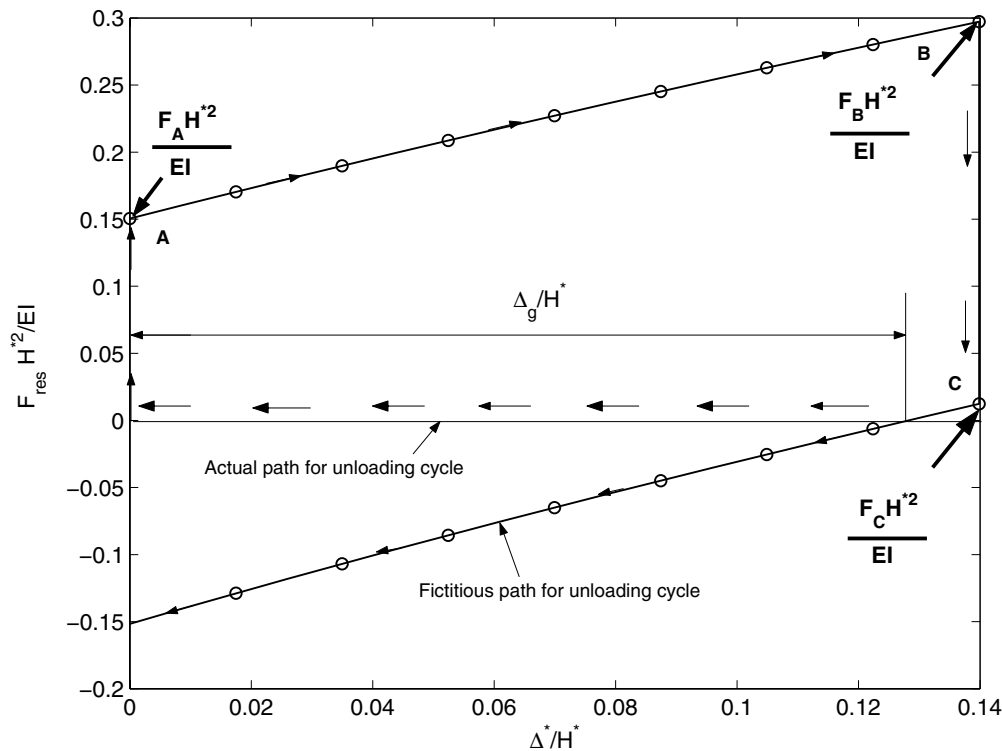


Figure 6

Relationship between

$$\frac{F_{res} H^{*2}}{EI} \text{ and } \frac{\Delta^*}{H^*}$$

during loading and unloading  $\Delta^*$  for a transition seal with  $R_s/H^*=8.9$ ,  $\theta=45^\circ$  and

$$\bar{m} = \frac{m^* H^{*2}}{EI} = 0.135.$$

$\Delta_g/H^*$  shows the position where fibers are "stuck", i.e., cannot completely recover from bending during unloading  $\Delta^*$

# Numerical Results and Discussion (Cont'd)

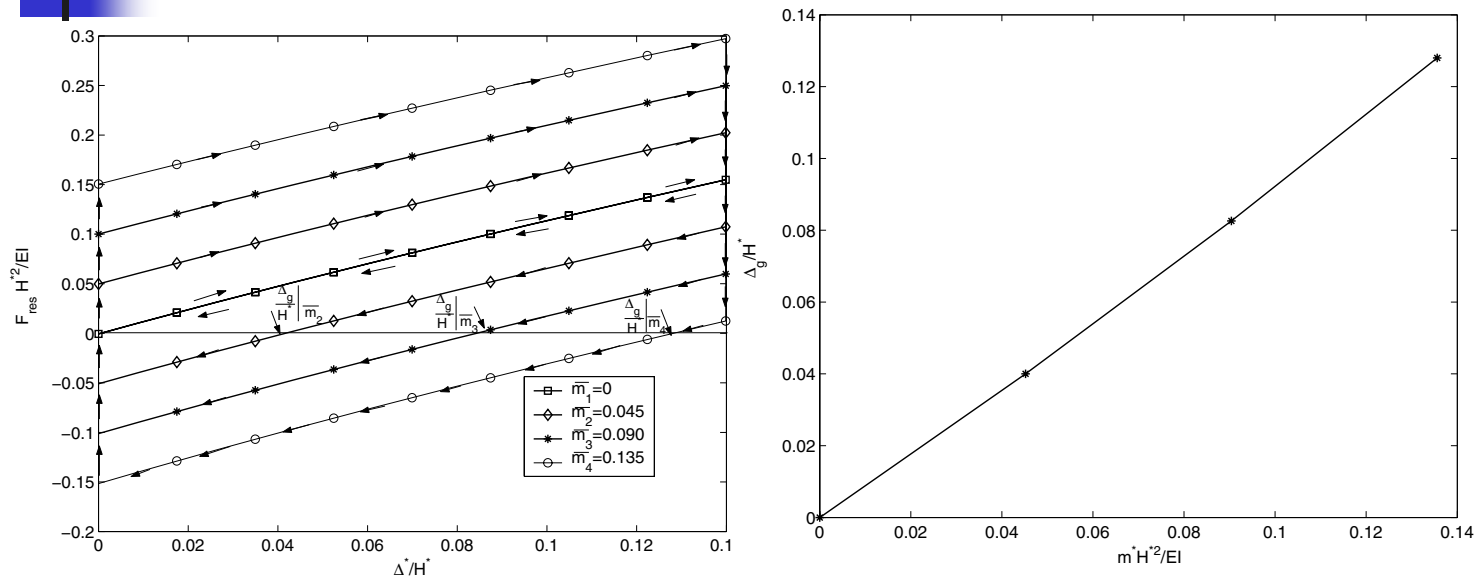


Figure 7 (a) Relationship between dimensionless contact force and dimensionless penetration depth for  $\bar{m} = 0, 0.045, 0.090, 0.135$ . (Results shown are for  $R_s/H^* = 8.9$ ,  $\theta = 45^\circ$ ; (b) relationship between  $\Delta_g/H^*$  and non-dimensional bending moment  $\bar{m}$

# Numerical Results and Discussion (Cont'd)

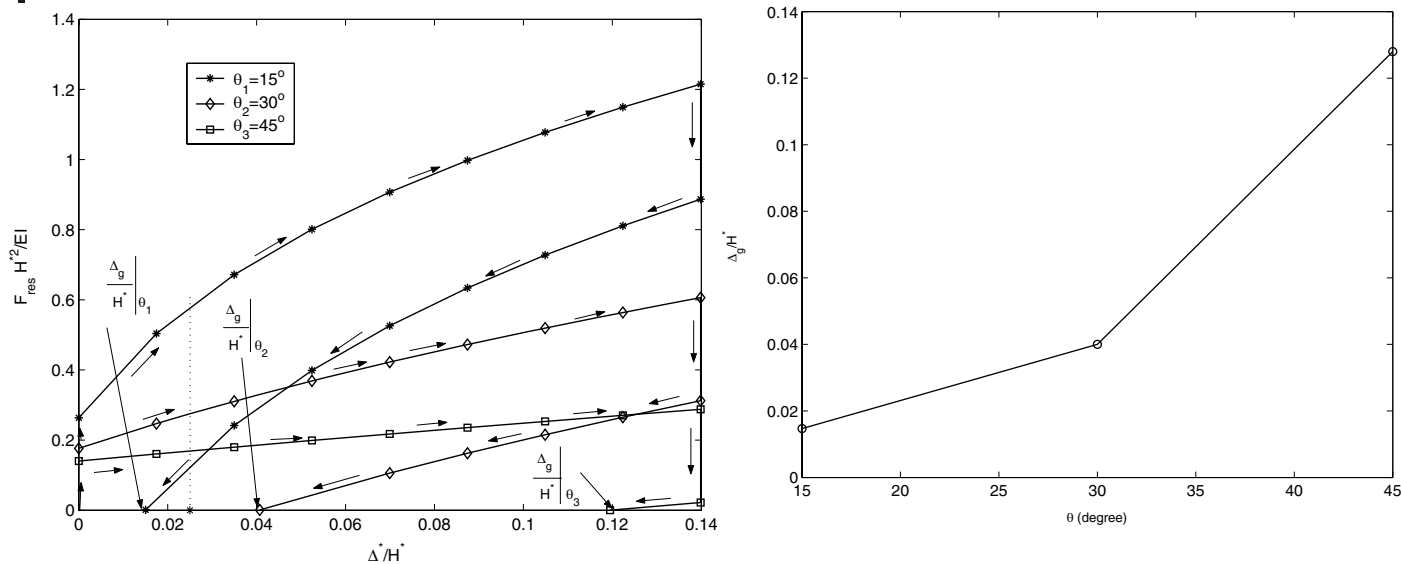


Figure 8 (a) Relationship between dimensionless contact force and dimensionless penetration depth for  $\theta=15, 30$  and  $45$  degrees. (Results shown are for a bristle with  $R_s/H^*=8.9$ ,  $\Delta_o^*=0$ ,  $\overline{m}=0.135$ ) and (b) Relationship between  $\Delta_g/H^*$  and bristle lay angle  $\theta$



## Conclusions/Summary

---

- The micro-moment can give rise to a delayed filament displacement as the shaft undergoes transient excursion and moves radially toward bristle pack (uploading).
- However, as the shaft returns back to its concentric position (downloading), the filament CANNOT completely recover from its deformed position and remains locked in an alternate configuration.
- Consequently, an annular gap is generated between the fiber tips and shaft surface, which promotes brush seal leakage and reduces turbo-machinery performance.



## Conclusions/Summary (cont'd)

---

- In general, for a given brush seal, the annular gap increases linearly as the micro moment  $m^*$  is increased.
- The brush seal having a shallowest lay angle ( $15^\circ$ ) results in the smallest annular gap, indicating that a brush seal design with shallow lay angle is least prone to hysteresis phenomenon, and can lead to improved performance.